LETTERS TO THE EDITOR

To the Editor:

A. R. Oroskar and R. M. Turian (O&T) give two equations relating the critical velocity (v_c) in pipeline flow of slurries to the volume fraction of solids (C), a second function of C, 1-C, the ratio of pipe diameter to particle size (D/d), a modified Reynolds number ($\overline{N}_{Re}$) which contains the product $D\sqrt{d}$, and the fraction of eddies having instantaneous vertical velocities greater than the terminal settling velocities of the particles (x) [*AIChE J*, **26**, 550 (1980).]. The factor x was estimated through a desperate analogy to molecular motion in gases and was stated to have always been >0.95 in the experiments they cited. The two equations were both power forms, one reached through semitheoretical arguments that filled most of the article's pages, the other found by regression, presumably multiple linear regression (MLR) applied to the logarithms of the factor values. The exponents obtained for the five factors and for d are tabulated below.

Factor:	C	1-C	D/d	$\overline{N}_{Re}$	x	d
Theoretical exponent:	0.5333	1.6000	0.533	0.0667	−0.53	−0.500
Regression exponent:	0.1536	0.3564	0.378	0.09	+0.30	−0.333

The dependent variable was presented as $v_c/\sqrt{gd(s-1)}$. Comparing this with the exponents found for D/d reveals two facts: (1) if the equations were multiplied through by $\sqrt{d}$, the actual exponent of d for v_c would be 0 or 0.167, and (2) because d was responsible for a large part of the total variation in the experiments, a big percentage of the coefficient of determination (R^2, not stated but roughly 0.4) is attributable to the inclusion of nearly equal powers of d in both sides of the equation. Had that not been done, the overall correlation coefficient would not have been significantly different from zero.

Many of the available MLR programs provide estimates of the standard errors of the coefficients, i.e., the exponents in the power form, which may be used to test the differences between the fitted and predicted values &/or to judge the statistical significance of the ones obtained by regression. Unfortunately, neither the standard errors nor the data needed to estimate them were presented by O&T, so we cannot make these tests.

The points of their Figure 7, reproduced below, follow the curve I've dashed in by eye more closely than the do the 45° line. This is a clue that the theoretical modeling has gone awry in one or more of its aspects. The violent disagreement between the theoretical and regressed exponents for C, 1-C and x semaphores the same mishap. Since the factor D/d was one of those most widely varied in the experimental work, its regression exponent, 0.378, is likely to have the smallest error of all. It, too, may well be significantly different from its theoretical counterpart.

Considering the very narrow range exhibited by the factor x and the crudeness of its estimation, I'd bet that the exponent 0.30 will not survive a test of the hypothesis that it is truly zero. The exponent of the Reynolds number, too, can be taken as zero. For those who might wish to estimate critical velocity from either of these equations (±43 or 52%), dropping these feeble factors, especially the elusive x, is a useful simplification.

The regression might have proved more instructive and useful if it had been run on the basic physical quantities measured: C, D, d, ρ_l, ρ_s and μ. The exponents of these might well have suggested other variable groupings more valuable for predicting v_c than either of the authors' ill-fitting equations.

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Reply:

I am pleased to respond to J. F. Carley's letter about our article "The Critical Velocity in Pipeline Flow of Slurries" by A. R. Oroskar and myself [*AIChE J.*, **26**, 550 (1980)]. It is true, as Carley has discovered, that our theoretical relation, Equation (41), for the dimensionless critical velocity, defined as $v_c/\sqrt{gd(s-1)}$, has a particle diameter dependence of $1/\sqrt{d}$. Accordingly, multiplying by $\sqrt{d}$ would eliminate d from both sides of the equation, as he points out. It happens that this fact, namely that v_c is a weak function or is independent of d, is consistent with the body of critical velocity correlations proposed to date, and is moreover supported by the data on slurry transport, regardless of Carley's presumptions in this respect. Reference to Table 1 of our article, which lists many of these correlations, confirms this fact. The two significant exceptions are the correlations by Newitt et al. and by Spells, which happen to do a particularly poor job of predicting the critical velocity. These two correlations are, moreover, deficient in a more fundamental respect; neither one depicts a dependence on slurry concentration, and the correlation by Newitt et al. predicts no dependence on pipe diameter as well. It should be noted, with regard to relationships which include it, that the particle drag coefficient, C_D, is approximately constant for the types of highly settling solids encountered in slurry transport.

Our theoretical expression, Equation (41), predicts the set of 357 critical velocity data listed in Table 2 of the article with an absolute average deviation of 19.5%. The maximum deviation for any data point is −103.5%, and only 24 data points have deviations exceeding ±50%. The meaning of such agreement between theory and experiment in the context of the problem of slurry transport will be discussed below. It is significant that our theoretical development results in a relationship containing essentially no adjustable constants, which quantitatively predicts the right sort of dependence on pipe diameter ($\sim D^{0.6}$) and accounts for the effect of concentration over a broader range than hitherto possible, particularly the practically important high range. Indeed it also does predict the observed dependence on particle diameter, which is very weak or nonexistent (see, for example, the empirical correlation of Durand, given in Table 1, which is based on the most extensive body of slurry flow data). These facts attest to the essential soundness of the ideas used in our theoretical development. It needs to be pointed out that Figure 7 in the article is a plot of the experimental values of $v_c/\sqrt{gd(s-1)}$ against the corresponding values predicted by Equation (41). Accordingly the 45° line we have drawn is meaningful. The dashed curve Carley has superposed on this figure, on the presumption it represents a better fit to the data, is really meaningless.

Our regression result, Equation (45), is obtained by curve-fitting a product form, consisting of the variable groupings identified by our analytical development, to the 357 data points. The dependence on d is likewise found to be weak, and is again consistent with the data. Also the dependence on pipe diameter ($\sim D^{0.47}$) is consistent with the approximate square-root dependence which is characteristic of slurry flow behavior. Equation (45) predicts the set of 357 data with an absolute average deviation of 15.4%, a maximum deviation of 89.0%, and with only 14 data points with deviations exceeding ±50%. Comparison of these results with those pertaining to our theoretical relation, and also Figures 7 and 8 of our paper, might suggest that on the average the regression result, Equation (45), does a somewhat better job of prediction than our analytical result, Equation (41). However, we felt the improvement is neither substantial nor uniform enough to warrant recommending the empirical expression in place of the analytical result. Furthermore, data covering a broader range of the variables than were available would be required to establish a broadly applicable regression correlation, and to test it critically.

Critical velocity data are very difficult to get, and are uncertain and equivocal because the critical condition is very difficult to discern. Indeed the instability in the flow, as the critical condition is approached, results in wide variations in the measured variables, particularly the concentration [see, for example, A. D. Thomas, *Int. J. Multiphase Flow*, **5**, 113-129 (1979)]. It takes a special lack of appreciation of the nature of slurry flow, and the intrinsic limits on reproducibility of data in such systems, to view the sort of agreement described above, with disdain. It takes an equal level of lack of knowledge of the problem to suggest that a gross regression on the direct physical quantities would result in better correlation. Would such an approach constitute stronger theory or stronger statistics than is contained in our work? I wonder what impels Carley to conclude that C, D, d, ρ_l, ρ_s and μ are the only pertinent variables governing the flow. Would stronger statistics compensate for the uncertainties inherent in data of this type? There surely must be other ways of predicting the critical velocity perhaps better than we have been able to do; Carley's statistical presumptions do not constitute one of those ways.

We need make no apologies for including the factor x. It is explicitly stated in the article that the analogy between turbulent eddies and molecules cannot be justified, and, at any rate, we point that its inclusion seems to make little difference in practice.

We need to emphasize that our critical velocity correlations were developed for the type of systems encountered in pipeline transport of slurries, which must consist of concentrated suspensions comprised of very large fractions of coarse solids. Otherwise the suspensions would be too thick to transport over long distances. Critical velocity and flow data for concentrated suspensions of *fine* particulate solids, typical of those encountered in coal refining, gasification and liquefaction are somewhat scarce, and are not subsumed by the present correlations. We are presently investigating such fine particulate suspensions, and although it is too early to predict their behavior we believe that the prevailing mechanisms are different, resulting perhaps in somewhat stronger dependence on particle diameter.

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